Fast and Globally Optimal Motion Planning with Distance-Geometric Formulations of Robot Kinematics

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Areas of Interest: Optimization, robotics, geometry.

Background and Motivation

Motion planning in complex environments is a fundamental capability for autonomous robotic manipulators. Ensuring that a planning algorithm has theoretical guarantees like completeness or optimality is challenging, especially for *redundant* manipulators[1] without closed-form inverse kinematics (IK) solutions. The state of the art is dominated by sampling-based algorithms with asymptotic guarantees. In practice, these methods suffer from the curse of dimensionality, and are often replaced or supplemented with approaches that use local optimization, heuristics, and reinforcement learning. While efficient, these practical solutions lack the formal guarantees required for high-risk manipulation tasks in safety-critical fields such as collaborative robotics, search and rescue, healthcare, and space exploration.

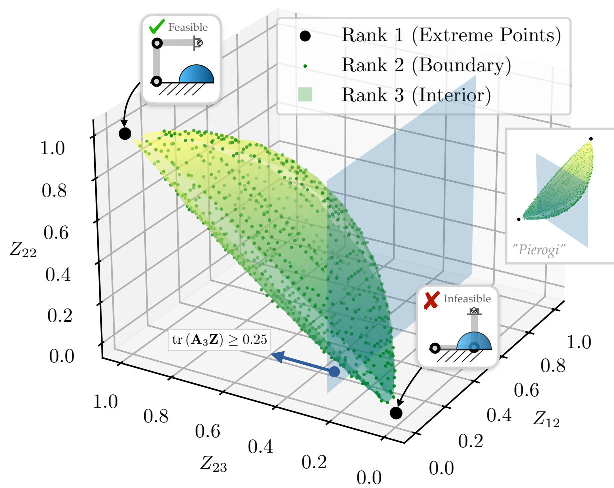

Figure 1: Spectrahedron representing the lifted feasible set for a simple 2-DOF planar manipulator IK problem (inset).

Recently, we have developed a variety of IK solvers for redundant manipulators in cluttered environments [1, 2, 3, 4]. These methods all rely on a *distance-geometric* description of robot geometry originally developed in [5]. By omitting the direct use of joint angles, this approach formulates kinematic problems as quadratically constrained quadratic programs (QCQPs). Using tools from convex optimization, these nonconvex QCQPs can be relaxed or "lifted" into convex semidefinite programs (SDPs). The feasible set of an SDP relaxation for a toy IK problem is shown in Figure 1. This procedure transforms the IK problem from a nonconvex program to a search for low-rank solutions to a convex feasibility problem (these are the black extreme points in Figure 1). Our results in [3] demonstrate that for a large class of manipulators, a heuristic search called convex iteration [6] recovers valid solutions with high probability. Additionally, in the lifted SDP domain, constraints which ensure avoidance of spherical obstacles become simple linear half-space constraints (e.g., the blue plane in Figure 1). This initial success for IK problems suggests that distance-geometric modelling may yield efficient path planning algorithms for redundant manipulators.

[1] These are robot arms with more degrees of freedom (DOF) than their workspace (e.g., the 7-DOF KUKA iiwa.)

Objectives

This project aims to extend our recent work on distance-geometric inverse kinematics to a variety of kinematic and motion planning problems for redundant manipulators. On the one hand, we are seeking a theoretical characterization of the effectiveness of convex iteration for distance-geometric inverse kinematics. This will involve studying the geometry of spectrahedra arising from the feasible set of redundant manipulators. In general, these sets will be higher dimensional and much more complex than the toy example of Figure 1, and understanding the distribution and properties of their extreme points is paramount. Based on insights from this investigation, we will implement an extension of CIDGIK which exploits our problem's unique geometry to efficiently solve motion planning problems as a sequence of linked IK problems. Ideally, this algorithm will possess completeness or optimality guarantees on a broad and practical class of problems. Over the course of the internship, a successful student will

1. study spectrahedral geometry and facial reduction techniques for distance geometry problems [7];
2. prove theorems which characterize the distribution of low-rank solutions to the IK problem in [3];
3. formulate path planning problems in the language of distance geometry; and
4. implement semidefinite programming solutions to distance-geomteric motion planning in the Julia language.

Qualifications

The Autonomous Robotics and Convex Optimization Laboratory (ARCO Lab) is looking for self-motivated researchers who are capable of working effectively as part of a small team. This project requires both a strong mathematical background and software engineering skills. Qualified candidates will have a strong foundation in linear algebra, familiarity with the theory of constrained optimization (e.g., Lagrangian duality, convexity, KKT conditions), and experience using software for modelling and solving optimization problems (e.g., `CVX`/`cvxpy`, `YALMIP`, or similar). Exceptional candidates will also have experience with one or more of the following topics: semidefinite programming (SDPs), the Lasserre hierarchy (also known as sum-of-squares optimization), real algebraic geometry (e.g., Groebner bases, positivstellensatz), inverse kinematics for robot manipulators, and writing optimization code in the Julia programming language. **Funding is available for exceptional candidates**.

References

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